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ABSTRACT

The basic problem which motivated this study is that the present universal support given the teaching of mathematics has led to a neglect of the justification question. That is, we are sometimes careless in presenting rationales for teaching mathematics and do not adequately consider the consequences of invoking particular rationales. The major purpose of the study was to identify and discuss answers to the question of why we should teach mathematics in order to provide some perspective for the present historical moment. Using the curriculum interest groups identified by Kliebard (i.e., humanists, developmentalists, social meliorists, social efficiency educators) as a framework for the research, the development of rationales for teaching mathematics was traced from 1890 to 1940. The historical data indicate that the humanist interest group, which emphasized the place of mathematics in the Western cultural heritage and the value of mathematics study for developing reasoning ability, faced a growing challenge from a developmentalist/social efficiency coalition which espoused a directly functional rationale for teaching mathematics. Based upon certain problematic responses by the humanists to the developmentalist/social efficiency coalition, present-day positions taken by the National Council of Teachers of Mathematics were analyzed. (Author)

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Why Teach Mathematics?

A Historical Study of the Justification Question

by

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> Well, three or four months run along, and it was well into the winter, now. I had been to school most all the time, and could spell, and read, and write just a little, and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could ever get any further than that if I was to live forever. I don't take no stock in mathematics, anyway.
>
> Huck Finn

The sine qua non of the curriculum field is the struggle to answer the question of what we should teach. As Herbert Kliebard (Note 1) has said, "If human mental capacities were so great that everything could be known or if the amount of knowledge available were so small that it could be known to everyone, then we would have no need for the study of curriculum" (pp. 2-3). Given the fact that human beings are faced with an almost unlimited amount of available knowledge and a limited capacity to know, it is of no small import that the subject area of mathematics has almost universal support for a place in the school curriculum. In fact, mathematics is so universally accepted as part of the elementary and secondary school curriculum that it is quite easy today to give little or no thought to its justification for being there.

It is unusual to find people like Huck, who "don't take no stock in mathematics, anyway." Generally, even those people who have not had the most pleasant experiences during their study of mathematics admit to the importance of the field. So, while doubts about content and method and the general quality of teaching are often expressed, the issue of whether mathematics should be taught is, in fact, not even an issue. Furthermore, it seems reasonable to assume that a mathematics educator would rarely, if ever, consciously challenge or weaken the claim of

mathematics to a place in the school curriculum. The basic problem which motivates this historical study is that because of the almost universal acceptance of the importance of mathematics and because of the tendency not to question that which provides the basis of one's livelihood, mathematics educators rarely address the question of why we should teach mathematics; and, when the question is addressed, the response is often limited and does not even accept as an option the possibility that mathematics as an area of study may not be as important as we assume.

Yet, as mathematics educators, we should be constantly raising the question of justification rather than merely responding to it if and when it is asked by others. The same question of justification which could lead to doubts about the place of mathematics in the school curriculum also provides the basis for the continual revitalization of our field. It is a question which we must continually address because there is no ultimate "right" answer and because the way we formulate and reformulate our answer will help us to address the questions of content, method, and quality of teaching that tend to be our greatest concerns.

When the answer to the question of why we should teach mathematics is viewed as an ongoing process, the value of historical perspective becomes apparent. We can learn from the struggles of our professional forbears as they addressed the same issue. The claim that we have not given the justification question enough attention does not imply that answers have not been given, that adequate answers have not been given, or even that eloquent answers have not been given. In fact, it is the major task of this study to discuss such answers -- whether inadequate, adequate, or eloquent--and to attempt to make a connection between various answers

and the way one might organize the form and content of the mathematics curriculum. Furthermore, this study is not intended to be the definitive response to the justification question. It is merely an attempt to raise the justification question, to link our struggles with those of our professional forbears, and to live up to the claim that our response to the question should be an ongoing process undertaken by all people who consider themselves mathematics educators. Even eloquent rationales, if not heeded, not read, and not critically examined, give no help to day-to-day life in classrooms.

The Value of Historical Perspective

Kliebard (1975) claims that "The values associated with viewing complex questions from an historical vantage point are not always clear cut or easy to state. Frequently, however, such an approach does serve to provide some sense of continuity and direction" (p. 39). That is, historical inquiry will not provide answers to contemporary curriculum problems in any direct way; but it does open up the range of questions that can be raised with respect to such problems. Without curriculum history, we, in a sense, have tunnel vision of current problems; we see them in a restricted framework. Historical perspective provides not only a sense of the range of possibilities, but also a fuller comprehension of the consequences of particular actions.

Applied to the question of justification in mathematics education, historical inquiry cannot be expected to provide the absolute and final answer to the question of why we should teach mathematics; but it can provide us with the range of answers which have been offered and help us

to better understand the consequences associated with invoking particular rationales. The process of "critically examining our heritage as a field of study" by carrying on "a dialogue . . . with our professional forbears" (Kliebard, 1968, p. 83) is as important to mathematics education as it is to the curriculum field, in general. This does not mean that our professional forbears necessarily had a better understanding of the consequences associated with invoking particular rationales and that we should follow their example. It does mean that we can learn from their struggles. In short, we cannot assume that history will be any less confused than the present. This does not diminish the value of historical inquiry; it merely cautions us to interpret our findings carefully.

The Framework for the Study

In the concluding chapter of the 1970 NCTM yearbook on the history of mathematics education, Phillip Jones claims that "the real issues in mathematics education are continuing over time, changing slowly if at all." That is,

> one of the lessons of history is that much about human beings changes only very slowly. Consequently activities, such as education, that are closely involved with the minds, drives, and emotions of human beings, change very slowly in their basic issues. In fact, the perspective of history often shows that what were thought to be significant changes in issues were more nearly mere variations in the relative emphasis or prominence accorded to them. (Jones, 1970a, p.455)

This commentary on the history of mathematics education is very much related to Kliebard's ideas about the development of the elementary and secondary school curriculum during the twentieth century. Kliebard (Note 2) characterizes mental discipline as representing "curriculum

orthodoxy in the nineteenth century" (p. 2). With the decline in the influence of mental discipline, the field of education moved into the twentieth century with four interest groups emerging, each of which hoped to determine curriculum policy in the years ahead. Kliebard labels these four groups as the humanists (still, in some ways, linked to the mental disciplinarians), the developmentalists, the social meliorists, and the social efficiency educators. Led by Charles W Eliot and William Torrey Harris, the humanists emphasized the importance of the traditional disciplines of knowledge as embodiments of the Western cultural heritage. They were pitted against three reform groups, one of which, the developmentalists, was led by Granville Stanley Hall, who argued that the curriculum should be based on the natural order of development in the child. The social meliorists, with roots in the writings of sociologist Lester Frank Ward, saw the school as an important instrument for improving society, in general, and the plight of suffering individuals, in particular. Finally, impressed with the studies conducted by Joseph Mayer Rice, the social efficiency educators advocated a "scientific" approach to developing a curriculum which would contribute to a smoothly functioning, efficient society, a curriculum through which each person would be prepared for his or her predetermined place in that society.

Jones's claim that the real issues in mathematics education change slowly, if at all, over time fits well with Kliebard's notion of the ongoing struggle between the four interest groups:

> The twentieth century was to become the arena where these four versions of what knowledge is of most worth and of the principal functions of schooling were presented and argued. No single interest group ever gained absolute supremacy, although general social and economic trends, the

> national mood, and local conditions and personalities affected the strength of each of these influences in different periods as the twentieth century progressed. In the end, what became the American curriculum was not the result of any decisive victory by any of the contending groups, but a loose, largely unarticulated, and not very tidy compromise. (Kliebard, Note 2, p. 23)

Kliebard's interest-group hypothesis is a valuable framework for this study. The views of each of the four interest groups described by Kliebard are based on different assumptions about the purposes of schooling. Hence, each interest group provides a different argument for why mathematics should be part of the school curriculum.

Kliebard describes the 1890's as crucial in the development of these interest groups. In fact, the 1890's can be seen as a "crucible for curriculum change":

> By the 1890's, nineteenth century society with its reliance on the face-to-face community was clearly in decline, and with the recognition of social change came a radically altered version of the role of schooling. No longer was the school the direct instrument of a visible and unified community. Rather, it became a mediating institution between the family and an increasingly distant and impersonal social order, an institution through which the norms and ways of surviving in the new industrial society would be conveyed. Traditional family life was not only in decline; even when it remained stable, it was no longer sufficient to initiate the young into a complex and technological world.
>
> With the change in the social role of the school came a change in the educational center of gravity; it shifted from the tangible presence of the teacher to the remote knowledge and values incarnate in the curriculum. By the 1890's, the forces that were to struggle for control of the American curriculum were in place, and the early part of the twentieth century became the battleground for that struggle. (Kliebard, Note 2, p. 1)

There is evidence to indicate that historians of mathematics education also see the 1890's as important (e.g.,DeVault & Weaver, 1970; Jones & Coxford, 1970a, 1970b; Osborne & Crosswhite, 1970). In fact, Jones (1970) claims that mathematics education as a separate and distinct professional area was established during the 1890's and at the turn of the century, with the work of J.W.A. Young at the University of Chicago and David Eugene Smith at Teachers College. Therefore, although mathematics was, obviously, a part of the American school curriculum before that time and even though rationales for the study of mathematics go back many centuries, the 1890's provide a convenient and valuable baseline for this study.

With the writings of individuals and groups found in journals, yearbooks, other books, committee reports, proceedings of meetings, personal correspondence, etc. as the data source, the relationship between general curriculum thought as expressed through these interest groups and the views held by mathematics educators is traced from the 1890's to the Second World War in order to identify rationales and the possible consequences of invoking particular rationales. 1940 is an arbitrary but reasonable date at which to conclude the historical inquiry. Not only were two significant reports on mathematics education published in 1940, one by the Progressive Education Association and the other done jointly by the Mathematical Association of America and the National Council of Teachers of Mathematics; one could also argue that the Second World War and the Cold War that followed helped to bring on the "modern era" in mathematics education. This does not mean that there was some clear beginning for the "modern era," and it is not meant to imply that

what has happened since the Second World War can be easily characterized as some coherent "modern era." (In fact, the interest-group hypothesis goes against both of these notions.) However, conducting the inquiry from approximately 1890 to 1940 is not unreasonable.

Conflict, Compromise, and Continuity: The Development of Justifications for Mathematics Education from the 1890's to the Second World War

> I earned my living at school slavery, teaching to children the most useless, the most disastrous, the most soul-cramping branch of knowledge wherewith pedagogues in their insensate folly have crippled the minds and blasted the lives of thousands of their fellow creatures -- elementary mathematics. There is no more reason for any human being on God's earth to be acquainted with the binominal theorem or the solution of triangles, unless he is a professional scientist, -- when he can begin to specialize in mathematics at the same age as the lawyer begins to specialize in law or the surgeon in anatomy, -- than for him to be expert in Choctaw, the Cabala, or the Book of Mormon. I look back with feelings of shame and degradation to the days when, for a crust of bread, I prostituted my intelligence to wasting the precious hours of impressionable childhood, which could have been filled with so many beautiful and meaningful things, over this utterly futile and inhuman subject. It trains the mind, -- it teaches boys to think they say. It doesn't. In reality it is a cut-and-dried subject, easy to fit into a school curriculum. Its sacrosanctity saves educationalists an enormous amount of trouble, and its chief use is to enable mindless young men from the universities to make a dishonest living by teaching it to others, who in their turn may teach it to a future generation.
>
> William J. Locke (quoted in Smith, 1911)

The history of justifications for teaching mathematics is a history of people creating, responding to, and acting back on general movements in society and in the field of education. It is a history filled with conflict and compromise, with people and their ideas competing and attempting to capture and mold the mood of the general public. But the history of justifications for teaching mathematics is also a history with a sense of continuity. There have always been and there will always be people like William J. Locke, people who, while recognizing the value of mathematics in our society, see its intensive study as essentially useless -- if not harmful -- for the average citizen. Not to be outdone by the "Lockeans," people like David Eugene Smith are convinced (and have always tried to convince others) of the value of studying mathematics. For Smith, the intensive study of mathematics is not merely useful to each citizen; its disciplinary (in the sense of "training the mind"), poetic, and even religious qualities make the life of each person who studies it richer, fuller, and more satisfying. Although the ideas are not always expressed as humorously as they are by Locke or as romantically as they are by Smith, these contrasting views are always present and provide a sense of continuity that underlies the conflict and compromise that come from attempts to create and adapt to societal and educational trends.

It is in being able to account for this dynamic interplay of conflict, compromise, and continuity that Kliebard's interest-group framework demonstrates its greatest appeal. The framework has limitations if one attempts to find in mathematics education clearly defined and always distinguishable interest groups. For example, there are few mathematics educators who could be called social meliorists; yet this may say more about mathematics educators than it does about the framework. In addition, the borderline between developmentalists

and social efficiency educators in mathematics education is not always clear (though there are clear examples of each group in the extreme); however, such problems of classification and distinction exist even in the curriculum field, generally. Indeed, given that a basic assumption in the interest-group framework is that the groups interact and evolve, we would expect classification problems to exist. The value of Kliebard's interest-group framework lies not as much in the ease with which one can classify a particular individual as it does in the extent to which it provides conceptually clear categories for analysis. In short, it does not explain everything; but it explains more than the simple traditional/progessive categories that tend to dominate historical analyses in education.

Changes in the School Population

Along with the clear growth in the school population during the early twentieth century came the common and frequently stated perception that the overall quality of the school population was diminishing. Many educators expressed views that might lead one to believe that the "army of incapables" described by G. Stanley Hall (1904) had, in fact, descended upon the schools. Based upon this widely held view of the diminishing intellectual capability of the school population, claims were made that school curricula needed to change in order to accomodate, control, and educate the changing school population.

In fact, the population of the school was increasing at a significant rate. In a recent issue of Phi Delta Kappan, Thomas James and David Tyack (1983) list figures on the the growth of the American high school that are typical of what one could find in a variety of sources. In

1890, 359,949 students were enrolled in American high schools; 6.7 per cent of 14-to-17-year-olds attended high school; and 3.5 percent of 17-year-olds graduated from high school. By 1910, the figures had increased to 1,115,398 high school students, 15.4 percent of 14-to-17-year-olds attending high school, and 8.6 percent of 17-year-olds graduating from high school. By 1940, the number of students attending high school was 7,123,009 -- almost twenty times larger than the 1890 total; 73.3 percent of America's 14-to-17-year-olds were attending high school; and 49 percent of 17-year-olds graduated from high school. Clearly, during the years covered by this study, the American high school experienced significant growth.

Given this significant growth, even if the percentage of "low" ability students (however one chooses to describe such students) remained the same, increasing the actual number of such students by such a significant amount would create a more visible "problem" (for some educators). However, the assumption that with an increase in the school population comes a decrease in the overall quality or intellectual capability of that population (that is, not just the actual number, but also the percentage of low ability students increases) is not unproblematic. What measure of intelligence would validate such an assumption? What would we accept as evidence that this was the case?

Thorndike, Cobb, Orleans, Symonds, Wald, and Woodyard (1923) attempted to provide evidence. In fact, they presented a more detailed argument than most writers in an attempt to establish the basic claim. Using figures for 1918, they stated that "almost one in three of the children reaching their teens in the United States enters high school"; furthermore, they stated that "the corresponding figure for 1890 is almost certainly not over one in ten" (p. 3). They went on to claim

that even though "we lack measures of the inborn capacities of the one in ten or eleven of a generation ago and have only very scanty measures of the capacities of the one in three today," "we have . . . excellent reasons for believing that the one in ten had greater capacities for algebra and for intellectual tasks generally than the one in three of today" (pp. 4-5). They based their argument upon the assumption that "we know that education is selective, that the correlation between native capacity and continuance in school to higher and higher grades is positive" (p. 5). Based on this assumption, they considered "the effect of what seems a reasonable degree of this selective force, say that represented by a correlation of .70" (p. 5) and, using a hypothetical population of 1000 students from each generation, concluded that the students of 1918 in fact had lesser capacities for intellectual tasks than students of 1890. The assumption about the relationship between native capacity and continuance in school is crucial to their argument, yet they stated in a footnote that

> The justification of this estimate would lead into highly technical arguments, unsuitable for presentation here. The actual data available are complicated by various factors; and their interpretation depends upon one's opinion of the relative shares of native capacity and the amount of education in determining the score made in such tests as the Stanford-Binet, the Army Alpha, and the Army Beta. (p. 5)

The point is that the assumption which is crucial to their argument is somewhat questionable; and, because of this, their argument, while providing interesting reasons for believing that students in 1890 had greater capacities for intellectual tasks than students in 1918, does not, in fact, provide "excellent" reasons. The basic claim remains problematic.

There are alternative scenarios that might explain the perceived change that was taking place in the school population. It is possible that the perceived change in the quality of students was a function -- not of actual changes in the students -- but of changes in their teachers. There were some educators who, in the case of mathematics, expressed a concern about the number of "unqualified" teachers of mathematics who had to be hired because of the dramatic increase in the number of students. Now, if what, how, and how well teachers teach has an impact on student achievement, such a change in the quality of teaching could lead to an overall decrease in student achievement; such a decrease in student achievement might lead one to incorrectly conclude that the students are not as capable as previous students.

There is a second alternative scenario. It may be that the biggest problem was not so much dealing with a significant change in the overall quality of the high school population but, instead, dealing with a significant change in the purpose and function of the high school necessitated by the tremendous growth it was experiencing. For students in 1890, secondary education may have been almost a ritual, a "rite-of-passage" to an esteemed place in society. In such a situation, there is little need to be concerned about individual differences in intellectual capability; just "getting through" is important. However, the "rite-of-passage" idea became less viable as more and more people were attending and graduating from high school; just "getting through" was no longer a necessary and sufficient condition for achieving an esteemed position in a dramatically changing society. In such a mileau, questions about individual differences become more important, at least in part because individual differences in capability help to "explain" the changing purpose

and function of high school attendance and graduation.

The point of presenting the two alternative scenarios is not to substantiate a claim that educators were wrong in assuming that the overall quality of students was diminishing as the school population was increasing. The point is that the situation was complex and cannot be easily explained. Furthermore, suppose that we could find and agree upon evidence to substantiate the claim that the overall quality of the school population was deteriorating; how would particular changes in the population "require" changes in the school curriculum? How would such changes interact with our nation's democratic ideals? While these are all interesting and important issues, it does not appear as though they formed the basis for educational discourse at the time the high school population was increasing so dramatically. Instead, it seems to have been generally assumed that the overall intellectual capability of the school population was, in fact, diminishing and that, because of this decline, something had to be done about school curricula. It is not surprising that such an attitude would have a significant impact on the teaching of mathematics.

Basic Trends in the Data

The four curriculum interest groups were in the process of emerging during the 1890's and at the turn of the century -- with each group at its own unique stage of development. Although they were the defenders of tradition and even though their hold on the curriculum was coming under attack, the humanists were the most highly developed interest group at the turn of the century. Unlike the humanists, who were the direct descendants of the mental disciplinarians, the three reform interest groups (i.e., the developmentalists, the social meliorists, and the social efficiency

educators) did not, in a certain sense, have the same stake in a particular content of study. That is, while the humanist interest group is defined -- primarily, but not exclusively -- by its recommended content, the three reform interest groups, in attacking the traditional course of study, proceeded -- at least at first -- on what Dewey (1938/1973) called "the basis of rejection, of sheer opposition":

> There is always the danger in a new movement that in rejecting the aims and methods of that which it would supplant, it may develop its principles negatively rather than positively and constructively. Then it takes its clew in practice from that which is rejected instead of from the constructive development of its own philosophy. (p. 20)

The reform interest groups began, then, with a "spirit of opposition." This does not apply equally to all of the reform interest groups or to all individuals affiliated with each group; nonetheless, it is a reasonable characterization.

Although the humanists are not labelled as a reform interest group and, in fact, represent the forces of tradition, it is important to recognize that they, too, rejected -- or at least modified -- some of the tenets of mental discipline and were also a part of a growing tendency towards reform in the curriculum. As expressed through the writings of Charles W. Eliot (e.g., Eliot, 1884, 1892a, 1892b, 1894, 1905; NEA, 1893), the transition from the mental disciplinarians to the humanists represents both a change in emphasis and an acceptance of a broader conception of the liberal arts. That is, while the development of reasoning ability remained important to many humanists, they gave more emphasis to the content of study itself, apart from the opportunity particular subject areas provided for improving reasoning ability. For Eliot, although the study of

various broad content areas was important, the importance of content always remained secondary to the development of reasoning power; it was William Torrey Harris, the other early leader of the humanist interest group, who really stressed the importance of passing on the Western cultural heritage (see, e.g., Harris, 1888, 1896; NEA, 1895).

Each of the interest groups espoused a particular set of criteria for justifying the selection of subject matter for the school curriculum. According to the humanists, mathematics should be taught because it is an important part of our Western cultural heritage and (at least for some of the humanists) because of its unique contribution to the development of an individual's reasoning ability. Although the developmentalists, the social efficiency educators, and the social meliorists did not reject the teaching of mathematics, it did not have the same place of importance given to it by the humanists. For the developmentalists (see, e.g., Hall, 1901, 1902), all subject matter became of secondary importance as the focus moved from bodies of knowledge to the individual student. Mathematics, then, should be taught only insofar as developmental experience shows it to be necessary; and the extent of an individual's interest and native endowment become the fundamental criteria in determining the extent to which a particular individual should study mathematics. Although Joseph Mayer Rice (1892, 1913) called for more than the "essentials" in school programs, his focus on using the present needs of society to justify what is "essential" in school curricula was to become a dominant theme for the social efficiency educators. Furthermore, whatever was to be taught should be taught with clear and specific goals in mind; and the extent of achievement of those goals should be scientifically measured. Mathematics

was clearly considered an "essential"; yet the mathematics program, generally, and the secondary mathematics program, particularly, were, in many ways, threatened by the social efficiency ideal. For the social efficiency educator, it was important that future scientists and mathematicians study mathematics in depth. However, for most people in an efficiently functioning society, very little mathematics beyond basic arithmetic was essential. Finally, as represented in the work of Lester Frank Ward (see, e.g., Ward, 1883, 1903), the social meliorist goals of equal opportunity and the fair distribution of extant knowledge were much more clear than the content of the school curriculum which would help achieve such goals. Given that Ward did not seem to think highly of mathematics or mathematicians, one cannot be certain of exactly where mathematics would fit in the social meliorist program.

Unfortunately for mathematics educators, as the twentieth century progressed, the social efficiency interest group -- with John Franklin Bobbitt and David Snedden taking over from Joseph Mayer Rice as standard bearers -- began to carry the day; particularly in the years between the two World Wars, the production metaphor embedded in the social efficiency ideal became the dominant lens for viewing educational issues. In the 1920's and 1930's, the mathematics education literature was characterized not simply by a defense of the role of mathematics in the school curriculum, but also by solemn warnings of a "crisis" in mathematics education. In most cases, the fear was not so much that mathematics would be eliminated as a subject area, but that it would be eliminated as a requirement for all students. The logical consequence of the social efficiency viewpoint was that not all people needed to study mathematics;

one needed to study only as much mathematics as was required for daily living or for one's vocation. Humanist mathematics educators like David Eugene Smith and Jacob William Albert Young were appalled at the thought of mathematics being looked upon only from a limited utilitarian (or directly functional) viewpoint.

Young (1906), for instance, provided a long list and description of justifications for teaching mathematics (pp. 13-46). His two main classifications of justifications related to the importance of mathematical subject matter and the importance of mathematics as a mode of thought. Under the former, he included the value of mathematics to civilization as a whole, to individuals in their daily activities, and to individuals in their chosen vocations. Furthermore, under this first category, Young saw the teaching of mathematics as important because mathematics is "so essentially characteristic of the human mind, so little influenced by environment, so uniformly present in every civilization" (p. 14) and because mathematics "is equally ingrained in nature, at least in nature as seen and interpreted by the human mind" (p. 15). But, according to Young, "the facts of mathematics, important and valuable as they are, are not the strongest justification for the study of the subject by all pupils. Still more important than the subject matter of mathematics is the fact that it exemplifies most typically, clearly and simply certain modes of thought which are of the utmost importance to everyone" (p. 17). More specifically, Young referred to the importance of "understanding statements, noting facts, and making inferences" (p. 46). Furthermore, "the skill gained from the study of mathematics . . . [is] available beyond the bounds of mathematics, whose form of reasoning is the ideal towards which all other reasoning strives; owing, however, to the simplicity and

narrow range of mathematical inferences, mathematics can only give the beginning of the requisite practice" (p. 46). Of fundamental importance to Young, then, was the idea that "mathematics properly studied tends to strengthen, does strengthen, the power of thinking independently and accurately" (p. 40), a position consistent with the basic humanist viewpoint.

The problem with a scenario that characterizes mathematics educators as defending tradition and, therefore, affiliating with the humanist interest group is that it leaves the impression that both humanists, generally, and mathematics educators, particularly, represent stagnant and unchanging ideas. This is clearly not the case. Just as this era was filled with innovative ideas of humanists such as Charles W. Eliot, ideas which were a fundamental part of the change taking place in the American curriculum, the 1890's and the turn of the century represent a significant era in mathematics education in terms of the issues being raised and the ideas being discussed. It is precisely the fact that mathematics educators -- even those steeped in tradition -- were advocating and responding to reform within the field of mathematics education that confounded the situation for mathematics educators. For, while the developmentalists and, particularly, the social efficiency educators were having an impact on the curriculum field generally, many mathematics educators saw their own efforts at "internal" reform as crucial. In the words of William Betz (1908),

> That one has a right to speak of a reform movement in the teaching of mathematics, must be evident to any but a very indifferent observer. The agitation in favor of better teaching is not confined to our own country;

> and its effects are felt in colleges, universities, secondary and primary schools, with almost equal force. It is unnecessary to recite . . . all the details of its origin and progress . . . . It is sufficient to say that since Prof. E.H. Moore's memorable address in December, 1902, "On the Foundations of Mathematics," an ever increasing number of teachers have become actively interested in the pedagogy of mathematics. (p. 625)

Major advocates of reform in the mathematics community at the turn of the century included Felix Klein in Germany (see, e.g., J.W.A. Young, 1903), John Perry in England (see, e.g., Perry, 1902), and Eliakim Hastings Moore in the United States (see, e.g., Moore, 1902). Through his presidential address delivered before the American Mathematical Society on December 29, 1902, Moore motivated a great deal of twentieth-century discussion about the "unification" of the areas within mathematics and of mathematics with the sciences. This is not meant to imply that all mathematics educators were ardent reformers. Smith, in particular, was opposed to most of the "unification" ideas (see, e.g., Smith, 1905). However, the presence of both "external" (in the general curriculum field) and "internal" (in mathematics education) reform movements clearly confounded the situation, because these external and internal movements did not occur in isolation from each other. It is the response by mathematics educators to this milieu of reform that provides the most interesting and potentially valuable outcome of this study. The following examples are illustrative of the various levels of response.

To go back to the example of William Betz (1908), while stressing the importance of internal reform -- which he attributed to E.H. Moore's address -- Betz attempted to place the reform movement in a wider context

in his discussion of "transforming influences." He listed five such influences, including "modern industrialism" creating "a great outcry for more practical school work," a "high school population more diversified than ever before," "certain revolutions in school curricula and methods of instruction," the "'new education,'" and "research in the domain of pure mathematics" (pp. 626-627). His discussion of the "new education" is particularly interesting:

> Its fundamental precept that all work must be arranged psychologically and adapted strictly to the child's power of comprehension, is eminently sound. But it has also given us the enriched curriculum, and the doctrine of interest which replaces all objective standards by the subjective attitude of the child. Unquestionably this means at once a distinct advance and a very real source of danger. The complaint is not infrequent that in many cases the young are learning to depend too much upon the inspirational powers of the teacher, that all real difficulties are carefully avoided, and that the very aim of all true education, to develop a strong character and to create self-activity and initiative, is thereby defeated. (p. 627)

Betz warned of "exaggerations" in the "new education" that "rob many otherwise excellent ideas of their legitimate influence and place upon them the stigma of the faddist" (p. 627). Betz is an interesting and informative example of the attempt on the part of some mathematics educators to look at the changes taking place in the educational world with a reasonably open mind. He did not wholeheartedly accept suggested reforms but, in regard to geometry in particular, he did say things like "The one-sided theory of mental discipline in the elementary course must go. The over-insistence on discipline for its own sake has done

incalculable harm to the interests of mathematics. It is well that we are beginning to realize this . . . . Our civilization craves action, and a subject that loses contact with life is doomed" (p. 630). He also asked the following question about the geometry course: "Should we not eliminate all that is not absolutely essential?" Does this make Betz a social efficiency educator?

It must be remembered that even Charles W. Eliot, the humanist, spoke of "shortening" the grammar school course. However, just as Eliot also called for a concomitant "enriching" of the grammar school course, Betz said about the geometry course that "the time gained by such omissions could be devoted to more significant topics, and would enable us to introduce at least a few interesting and valuable applications" (p. 631). Using ideas and terms similar to those of the most ardent social efficiency educators, Betz produced a far different message, one based upon the assumption that "never has the unique importance of mathematics been more generally recognized, nor has there been more mutual sympathy and cooperation" (p. 633).

No, Betz was not a social efficiency educator; but both the developmentalist and social efficiency viewpoints certainly had an impact on him. He was, quite simply, a mathematics educator attempting to respond to his era in his own unique way. A crucial point is that he viewed suggested reforms with a basic faith in the value of teaching and learning mathematics; the same could not be said of some of the social efficiency educators whose terminology he sometimes used.

While Betz was attempting to evaluate suggested reforms with an open mind, it appears as though some mathematics educators attempted to use the language of the developmentalists and social efficiency

educators to advance their own internal reform preferences. The best example of this comes from some of the advocates of unified mathematics. The extent to which this use of external reform rhetoric had problematic consequences for justifying the teaching of mathematics is an interesting part of the history of mathematics education.

Using adjectives like "correlated," "fused," "unified," and "parallel," certain mathematics educators, predominantly at or near the University of Chicago, advocated "breaking down the walls" between mathematics and other subject areas and between the various areas within mathematics itself. These educators were definitely not all arguing for the same thing; in fact, there were significant differences of opinion within this group (see Sigurdson, 1962). Works by Breslich (1916, 1933, 1935); Breslich, Schorling, Wright, and Irwin (1916); Hawkes (1911, 1912); Myers (1907); Reeve (1916, 1920. 1922); and J.W.A. Young (1907, 1912) give some indication of the kinds of discussions taking place (though these by no means reflect all that was being said about unified mathematics). The crucial point for this discussion can be brought out by looking a bit more closely at what Reeve (1916) was saying.

Reeve (1916) began by speaking highly of H.C. Morrison's (1914) chapter in the Thirteenth Yearbook of the National Society for the Study of Education, Part I, a chapter which reflected a mixture of developmentalist and, particularly, social efficiency thought applied to mathematics. According to Reeve,

> No one who reads Mr. Morrison's article will feel that he is attempting to discredit the place of mathematics as a part of the child's educational equipment. On the other hand, his attitude is friendly and constructive -- this, it seems, is one good reason why all mathematics teachers should read what he has to say. (p. 204)

Not all mathematics educators described Morrison's discussion in quite the way as did Reeve; Minnick (1915), for instance, characterized Morrison's work as a "severe criticism against present practices in mathematics" (p. 81).

Reeve then mentioned criticisms of the teaching of mathematics made by David Snedden and Abraham Flexner. He did not accept all of Snedden's and Flexner's criticisms, but he did use their criticisms of mathematics education to support his own views. In particular, he responded to Flexner as follows:

> It may not be a hard task for teachers of mathematics to convince Dr. Flexner that many of the faults of which he speaks are now apparent to the progressive teachers of mathematics and that in many cases the attempts to remedy existing conditions have been not only intelligent, but noteworthy and in some cases in line with nearly all the reform Dr. Flexner suggests. (p. 205)

Furthermore, Reeve said that he was in "perfect agreement" with "some of Dr. Flexner's statements." "It is to be hoped," said Reeve, "that mathematics teachers will not sit by and permit all of the criticisms in the future to be made by outsiders" (p. 206). In response to Flexner's belief that "the course in mathematics should include 'nothing for which an affirmative case cannot now be made out,'" Reeve replied: "This is exactly what teachers of mathematics who are really interested in the future are trying to do" (p. 207).

Reeve's answer to these critics, although he said it would not meet all of their "demands," was "some scheme of unifying high-school mathematics . . . keeping in mind all the while that increased efficiency in the technique of teaching must be looked after at all times" (p. 207). Two examples of Reeve's blatant attempt to use the rhetoric of the "outside"

critics to further his own internal reform can be seen in the description of his unified program:

> A decided advantage of such unification in the first year is that it gives the child a broader knowledge of the nature of the field of mathematics so that he is able to see better what lies in store for him if he goes on. And in the second place, if he does not continue, he has what the writer thinks is a better asset from a social standpoint. This second item it seems is what most people are demanding of the subjects in the curriculum, viz., that they function socially. (p. 209)
>
> In the third year the unification can be made as easily (so far as the subjects themselves are concerned) and with as much added interest and value in economy of time and efficiency of effort. (p. 211)

Reeve made a particularly interesting comment near the end of the article, a comment in which he more or less admitted that he was linking a reform movement with its own history in mathematics education with the reforms suggested by the "outside" critics:

> Finally let it be remembered that these suggestions are not considered as final, and that they are not given merely in the hope that something may be done to satisfy the cry for reform in the teaching of mathematics (which is in many respects legitimate) but to try to remedy some of the weaknesses of the system of which we were aware long ago, and also to increase the mathematical growth of the pupils who go out from our classes so that their knowledge may function most efficiently. (pp. 211-212)

The basic issue to be raised is the extent to which this somewhat forced marriage of external and internal reform movements had unfortunate consequences for attempts to justify the teaching of mathematics. The fact is that, given its previous position of stature, the place of mathematics in the school curriculum was not as strong as it had once been.

Furthermore, to a great extent, it was the social efficiency interest group that played the major role in calling the place of mathematics in the school curriculum into question. Clearly, Reeve and other mathematics educators who attempted to use the rhetoric of the critics to their advantage cared a great deal about mathematics education, generally, and about unified mathematics, specifically. Given their era and the fact that unified mathematics, while prominent in discussions, did not seem to be prominent in classrooms, their response was not unreasonable. However, as one looks at what a number of mathematics educators were beginning to perceive as a growing "crisis" in mathematics education (a crisis that, ironically, a somewhat changed Reeve did his best to make people more aware of with his editorials in the Mathematics Teacher during the late 1930's), one must at least bring up the possibility that the acceptance of the rhetoric of the critics without fully considering the long-term consequences of such an action might have played a role in exacerbating the "crisis" that mathematics educators were beginning to perceive. It may be that in any dispute, the language used to describe the situation provides a particular kind of lens for people to look through in attempting to make decisions about what should be done. Reeve was not unique in his use of the language of the social efficiency educators in his discussions; he was, however, somewhat unique in the extent to which he accepted the criticism and attempted to use it to further his own ideas. One could not claim that this attitude "caused" a crisis in mathematics education. At the very least, however, it did not hurt the cause of the social efficiency educator.

There were still other levels of response by mathematics educators. William Betz provides another good example, particularly in light of his 1908 response discussed earlier. By 1913, Betz was still looking

at all suggested reforms with an open mind. Like many others, he claimed that "it is an open secret that our present policy of extending every educational opportunity to the masses has brought about a further lowering of standards. This may be unavoidable, but it does not make the fact less serious" (Betz, 1913, p. 220). Although he felt that the "demand" for "efficiency" in the high school "should be welcomed enthusiastically by all teachers," he again warned of "loose thinking":

> In many of the current efficiency arguments there is an element of charlatanism, of gross materialism, and of loose thinking. Some efficiency experts would have us think that the work of a school can be gauged like the output of a factory. It should not be forgotten that the finest and most lasting results of good teaching cannot be tested by the methods of the counting-room or measured in dollars and cents, and in fractions of a second. (pp. 222-223)

Betz, in a sense, was attempting to distinguish betwen efficiency as the ability to use acquired knowledge and efficiency as economy of time and effort and the elimination of waste.

Unlike his comments in the 1908 article reported earlier, Betz even warned of the dangers of overreacting to mental discipline experiments. "Contrary to the usual impression conveyed by superficial alarmists," commented Betz in 1913, "it may be said confidently that 'we have neither proved nor disproved the formal discipline dogma'" (p. 224).

In his discussion of "the aims of mathematical teaching," Betz claimed (again using "efficiency" in the sense of the ability to use acquired knowledge) that the aims "center around the two ideas of insight and efficiency, which may also be described by the terms impression and expression, or by form and content. The one is subjective, the other objective." He then described the "five important aims of mathematical

teaching," which he classified under symbolism, applications, the function concept, space intuition, and logic (p. 226). About the aims of mathematics, Betz concluded that

> It is very doubtful whether any other high school subject can produce equally cogent and permanent reasons for being included in the curriculum. This becomes all the more apparent if it is observed that each of these five aims is essential from either a purely ideal or a narrowly utilitarian standpoint. To borrow a happy phrase of Professor D.E. Smith, mathematics is to a remarkable degree "potentially practical." It may be said without hesitation that if the opponents of mathematics should ever succeed in abolishing it from the curriculum, public opinion would demand its restoration within a year. Mathematics meets a fundamental human need. Finally, let it be remembered that ours is an age of mental unrest and uncertainty. To acquaint the minds of the young at this critical period with reliable standards of intellectual certainty is a service no less real because it is often overlooked, a service that endows mathematics with the highest moral value. (p. 227)

Like many other mathematics educators, Betz wrote with a spirit of compromise, saying that "The mathematical curriculum of the future must meet the wants of all types of students. For some of these types only actual experiment can determine the most desirable selection and sequence of topics " (pp. 229-230). But he also said that the "five aims cannot be fully realized unless every student in the general course is required to become familiar with the elements of algebra, of plane and solid geometry, and of plane trigonometry " (p. 230). In defending the latter claim, Betz, to a certain extent, turned the social efficiency educators' directly functional rationale back on them:

> If any one should consider this an unreasonable demand, let him make a brief study of the European requirements. If America wishes to compete with Europe along industrial, technical, or scientific lines, it cannot ignore the increased educational opportunities given to European secondary students. (p. 230)

Admitting that mathematics was important to our society as a whole, social efficiency educators nonetheless felt that this did not mean that all students had to study mathematics in depth. It was, they thought, essentially useless to the day-to-day activities of most people. Social efficiency required that only a few people study mathematics in depth. (One must remember here that even the definition of "in-depth study" would be different for a mathematician and a social efficiency educator.) However, in bluntly linking our future national security to the argument, Betz was suggesting that -- even though it would be beneficial for each student as an individual -- as many people as possible should take a reasonable amount of mathematics in the high school in order to make certain that each could play a role in the competition with other nations and that all potential mathematicians and scientists could be discovered. In one sense, it was an argument against the belief of certain social efficiency educators that future societal roles could be clearly predetermined. In another sense, it was an argument for a certain kind of "social efficiency."

Finally, the National Council of Teachers of Mathematics, which was established in 1920, is itself an example of a response to the reform mileau. In the first issue of the Mathematics Teacher published by the National Council (it was previously the official journal of the Association of Teachers of Mathematics in the Middle States and Maryland), the following "reasons for the National Council" are provided:

> First, it will at all times keep the values and interests of mathematics before the educational world. Instead of continual criticism at educational meetings, we intend to present constructive programs, by friends of mathematics. We prefer that curriculum studies and reforms and adjustments come from the teachers of mathematics rather than from the educational reformers.
>
> Second, it will furnish a medium through which teachers in one part of the country may know what is going on in every other part of the country. Significant reports and studies and experiments will be given wide circulation through the official journal. Otherwise they would be known only locally.
>
> Third, the Council through its journal will furnish a medium of expression for all of the teachers of the country. Thus, a feeling of solidarity will be aroused. All teachers of mathematics will know that they are members of one family, working together under a common leadership.
>
> Fourth, the Council will help the progressive teacher to be more progressive. It will also arouse the conservative teacher from his satisfaction and cause him to take a few steps ahead.
>
> Fifth, the splendid work of the National Committee on Mathematical Requirements will be conserved and extended as time and new conditions may show the need.
>
> Sixth, the Council should receive the support of every teacher of secondary mathematics because it intends to maintain a journal whose editors and writers are real class room teachers. In fact, the teachers themselves are the owners of the journal, if they will but join the Council. (Austin, 1921, p. 3)

The first and fourth reasons are particularly interesting and informative. The NCTM was, at one and the same time, leery of "educational reformers" and supportive of "progressive teachers." Yet, given the diversity of suggested reforms inside and outside of mathematics education, these reasons are not necessarily contradictory. It is

especially interesting that Harold O. Rugg, generally characterized as a "progressive," was the chairman of the committee that put together the first constitution of the organization; and he also served as its first vice-president. It must be remembered that, fundamentally, the NCTM was formed in response to attacks on high school mathematics. To quote again from the introductory article:

> Mathematics courses have been assailed on every hand. So-called educational reformers have tinkered with the courses, and they, not knowing the subject and its values, in many cases have thrown out mathematics altogether or made it entirely elective . . . . To help remedy the existing situation the National Council of Teachers of Mathematics was organized. (Austin, 1921, pp. 1-2)

It appears as though the NCTM defined "progressives" as supporters of mathematics and labelled their foes as "so-called reformers."

Within the limits defined by acceptance of external reform ideas at one extreme and continued support for the humanist justification at the other (and there was, in fact, consistent and continual support, not just for the humanist justification, generally, but even for mental discipline, specifically), the examples which have been discussed are illustrative of the various levels of response of mathematics educators to suggested reforms. The examples are not exhaustive of all levels of response, but they do provide a reasonable picture of the way in which mathematics educators attempted to justify the teaching of mathematics in a changing school and society.

Conclusion

In particular, an understanding of Betz's "twist" of the social

efficiency rationale and Reeve's use of the rhetoric of "external" reformers might provide valuable perspective for today's mathematics educators. Regarding Betz's "twist" of the social efficiency rationale, one can find other examples of mathematics educators (and, at times, general educators) linking school mathematics programs to our national security. In particular, it may be that the "modern math" movement of the late 1950's and early 1960's represents an interesting -- and perhaps problematic-- mixture of the humanist rationale with this revised social efficiency ideal. More recently, Stephen S. Willoughby, the president of the National Council of Teachers of Mathematics, began his reign as president with a brief piece in the NCTM News Bulletin entitled, "In Support of Military and Economic Strength" (Willoughby, 1982). His introduction is particularly revealing:

> The next two decades will be critical for the United States. Our ability to defend ourselves militarily is in doubt. Our presence in the marketplace for manufactured goods has already been seriously compromised in such important fields as communications and transportation, and we are rapidly losing our grip on areas such as computer and space technology. It is entirely within the realm of possibility that the United States will manage to become a second-rate power in the next 20 years.
>
> There are many reasons for this decline. Some such causes are broadly economic or political and must be addressed in those domains. However, a remarkable fact that is hard to ignore is the relatively great emphasis placed on the teaching of mathematics, engineering, and the natural sciences in both Japan and the Soviet Union -- our major competitors, respectively, in economic and military strength. (p. 2)

Now, the issue here is not whether Willoughby is right or wrong about his facts; nor is it whether he is right or wrong in using the rhetoric of national security to emphasize the importance of school mathematics programs. The point is that one must be careful about

using such rhetoric because it brings with it a whole "bag" of consequences that must be carefully considered and dealt with, consequences that flow from the social efficiency ideal which is being tacitly accepted. Unless we are careful to explain otherwise, the use of such rhetoric implies that our greatest concern is with finding the talented few who can bolster our national security and that more people should study more mathematics in order to assure that the talented few are found. What does this say about the value of the study of mathematics for the "average" citizen? Furthermore, is its value to the talented few only related to its importance in their future vocations? The fact is that the basis of the social efficiency ideal is to determine future societal roles as soon as possible and train people for their specific roles. The claim here is not that Willoughby in fact supports these consequences of the social efficiency ideal. But if he does not, he must be careful in making his argument. If we believe that there are multiple justifications for teaching mathematics, we must clarify how rationales interact in order to avoid possible contradictions in our argument. In short, when we invoke a particular rationale, we must deal with all its consequences, not just those we want to see.

This same problem exists in a present-day situation in which, as in the case of Reeve, external reform rhetoric is used to support internal reform. Consider An Agenda for Action, the highly publicized statement of recommendations for school mathematics programs in the 1980's presented by the National Council of Teachers of Mathematics. In essense, the authors of the Agenda have accepted the rhetoric of the "back-to-basics" reformers and attempted to fit their internal reform (the primary recommendation being to make problem solving the focus of school mathematics

programs) within this "external" reform rhetoric.

Generally, when people talk about basic skills in mathematics, they seem to be referring to computational skills with whole numbers, fractions, and decimals. Evidence for this is the fact that most of the minimum competency testing programs around the country have operationally defined basic skills in mathematics in this way. (As of 1980, at least thirty-eight states had mandated minimum competency testing programs. See Pipho, Note 3.) The NCTM has accepted the "back-to-basics" rhetoric but has also attempted to "redefine" the term "basic skills," borrowing the view of basic skills expressed by the National Council of Supervisors of Mathematics (1977). Ten basic skill areas are listed in the Agenda, including problem solving; applying mathematics in everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computational skills; geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict; and computer literacy (NCTM, 1980, pp. 6-7). Essentially, the NCTM is saying that it, too, supports the emphasis on basic skills, as long as everything it wants in a mathematics program is called a "basic skill."

We have seen that the acceptance of external reform rhetoric to advocate internal reform can lead to problematic consequences, because such an action implicitly supports the external reform (with all its consequences) and has the potential to distort the true meaning of the internal reform. This is true in the case of the Agenda when the NCTM characterizes problem solving as both a unifying theme for the mathematics program and as one of ten "basic skills" to be learned. If this is not contradictory, the NCTM at least diminishes its argument for permeating

the entire mathematics program with problem solving by characterizing what appears to be a general approach to the teaching and learning of mathematics as simply one of ten technical skills to be learned. The claim here is not that the NCTM would want its argument characterized in this way; the claim is that its use of "back-to-basics" rhetoric creates some problematic consequences that must be dealt with.

The results of this historical inquiry do not "tell" us what to do in the two examples which have been described; each historical situation is, in a certain sense, unique. But this study does show that we must be careful in invoking particular rationales and reform rhetoric in supporting our own point of view. Different rationales arise from different assumptions about what mathematics is and what it means to be an educated human being; hence, these rationales have different implications for the mathematics curriculum. Invoking a particular rationale for teaching mathematics should mean that the assumptions behind the rationale and the implications are recognized and accepted. That is, for any particular mathematics curriculum, there should be a relationship between the reasons given for teaching mathematics and the form and content of the curriculum. Unfortunately, this vital connection does not always exist--precisely because the universal support given the teaching of mathematics allows one to give limited importance to the justification question and to move on to the supposedly more vital concerns associated with making curriculum recommendations. Hence, though people still feel compelled to list one or more rationales for teaching mathematics, the leap between rationale and recommended curriculum is too often carelessly made.

A particular justification for teaching mathematics does not imply a clear, obvious, and unique form and content for a mathematics

curriculum; but it does guide one in making decisions. Dewey, perhaps better than anyone, helps us understand the necessary care with which one must make connections between rationales and curricula. In How We Think, Dewey (1910), in a certain sense, responded to both the advocates of a directly functional rationale and those who are opposed to a limited utilitarian point of view. To those with a "practical" orientation, he said:

> There is such a thing, even from the common-sense standpoint, as being "too practical," as being so intent upon the immediately practical as not to see beyond the end of one's nose or as to cut off the limb upon which one is sitting. The question is one of limits, of degrees and adjustments, rather than one of absolute separation. Truly practical men give their minds free play about a subject without asking too closely at every point for the advantage to be gained; exclusive preoccupation with matters of use and application so narrows the horizon as in the long run to defeat itself. It does not pay to tether one's thoughts to the post of use with too short a rope. Power in action requires some largeness and imaginativeness of vision. Men must at least have enough interest in thinking for the sake of thinking to escape the limits of routine and custom. Interest in knowledge for the sake of knowledge, in thinking for the sake of free play of thought, is necessary then to the emancipation of practical life -- to make it rich and progressive. (pp. 138-139)

In his defense of his ideas about "occupations" in the classroom (it must be remembered that Dewey did not use the word in the vocational sense as we sometimes think of it), Dewey responded to "opponents of direct and useful activities":

> Just as the opponents of play in education always conceive of play

> as mere amusement, so the opponents of direct and useful activities confuse occupation with labor. The adult is acquainted with responsible labor upon which serious financial results depend. Consequently he seeks relief, relaxation, amusement. Unless children have prematurely worked for hire, unless they have come under the blight of child labor, no such division exists for them. Whatever appeals to them at all, appeals directly on its own account. There is no contrast between doing things for utility and for fun. Their life is more united and more wholesome. To suppose that activities customarily performed by adults only under the pressure of utility may not be done perfectly freely and joyously by children indicates a lack of imagination. Not the thing done but the quality of mind that goes into the doing settles what is utilitarian and what is unconstrained and educative. (p. 167, emphasis added)

As an illustration of the difficulty of making clear connections between rationales and curricula, consider advocates of "modern math" who suggested using "intermediate" or "developmental" algorithms to help children understand the mathematics behind the final, efficient computational algorithm. They did not anticipate the fact that certain teachers would look upon these intermediate algorithms as a focus for more drill work, and not as an aid to understanding, which completely distorted the purpose of this part of the curriculum. To repeat what Dewey said, "Not the thing done but the quality of mind that goes into the doing settles what is utilitarian and what is unconstrained and educative." This should not diminish our hope of ever developing a mathematics program that, in its implementation, reasonably reflects

our ideas about what mathematics is and why it should be taught. It should make us realize that, if we ever hope to make such a connection, we need to give the justification question much more careful thought. The justification question is one which must be addressed by all subject area specialists and by curriculum theorists as we all continue to struggle with the issue of why we should teach one thing rather than another.

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Abstract

The basic problem which motivated this study is that the present universal support given the teaching of mathematics has led to a neglect of the justification question. That is, we are sometimes careless in presenting rationales for teaching mathematics and do not adequately consider the consequences of invoking particular rationales. The major purpose of the study was to identify and discuss answers to the question of why we should teach mathematics in order to provide some perspective for the present historical moment. Using the curriculum interest groups identified by Kliebard (i.e., humanists, developmentalists, social meliorists, social efficiency educators) as a framework for the research, the development of rationales for teaching mathematics was traced from 1890 to 1940. The historical data indicate that the humanist interest group, which emphasized the place of mathematics in the Western cultural heritage and the value of mathematics study for developing reasoning ability, faced a growing challenge from a developmentalist/social efficiency coalition which espoused a directly functional rationale for teaching mathematics. Based upon certain problematic responses by the humanists to the developmentalist/social efficiency coalition, present-day positions taken by the National Council of Teachers of Mathematics were analyzed.